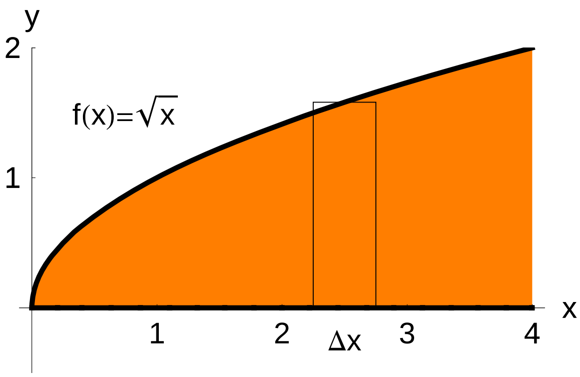
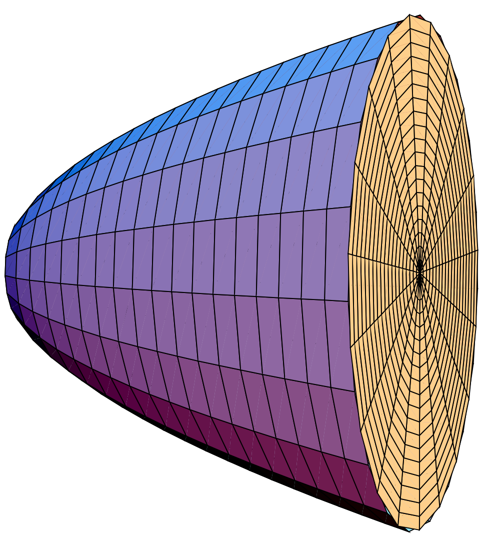
AP Calculus
Review: Volumes of Solids of Revolution

Let ***R*** be the region in the first quadrant bounded by the graph of f(x) = $\sqrt{x}$, the x-axis, and the lines x = 0 and x = 4 as shown below. We wish to find the volume of the solid generated when region ***R*** is revolved about the x-axis.

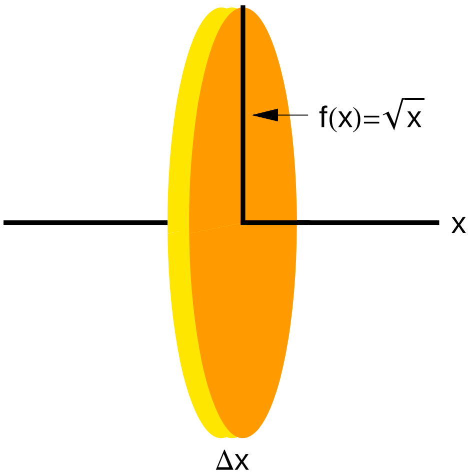


The solid of revolution looks like this:



Note that we have one rectangular element on the interval [0, 4] drawn on the original graph. The height of this rectangle is y = f(x) = $\sqrt{x}$ and the width is **Δx.**

When this rectangular element is revolved about the x-axis it will "sweep" out a disk shape.



The radius of the disk will be the height of the function, y = f(x) = $\sqrt{x}$, and the height of the disk will be **Δx.** So the volume of the disk is given by

$$V_{\text{disk}} = \pi\, r^2\, h = \pi\left(\sqrt{x}\right)^2 \Delta\text{x} = \pi \text{ x } \Delta\text{x}$$

If we approximate the volume of the solid with a finite number of disks "stacked" next to each other we get

$$V_{\text{solid}} \approx \sum_{i=1}^{n} \pi \text{ x } \Delta\text{x}$$

So the exact volume of the solid can be found by adding up an infinite number of infinitely thin disks all of whose heights are y = f(x) = $\sqrt{x}$. The greater the number of disks, the thinner each disk becomes. So, if we take the limit of the Riemann sum above as the number of disks goes to infinity (or as **Δx** goes to zero) we obtain the following definite integral.

$$V_{\text{solid}} = \lim_{n\to\infty} \sum_{i=0}^{n} \pi\, x\, \Delta\text{x} = \pi \int_0^4 x\, dx$$

Evaluating this integral using the FTC we obtain

$$\pi \int_0^4 x\, dx \;=\; \pi\, \frac{x^2}{2}\, \Big|_0^4 \;=\; \pi\left[\frac{4^2}{2} - 0\right] \;=\; 8\,\pi$$