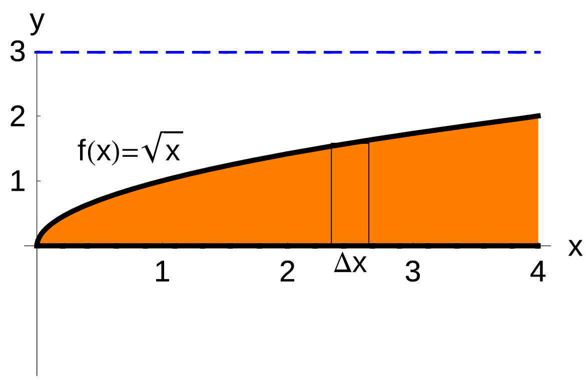
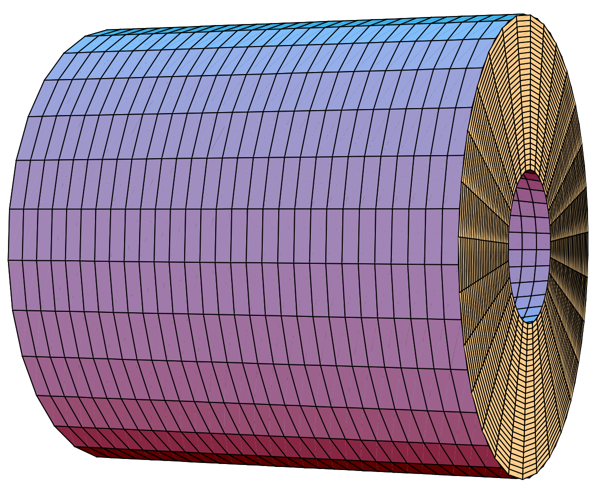
# AP Calculus
# Review: Volumes of Solids of Revolution, revolving about a line parallel to the x-axis

Let ***R*** be the region in the first quadrant bounded by the graph of f(x) = $\sqrt{x}$, the x-axis, and the lines x = 0 and x = 4 as shown below. We wish to find the volume of the solid generated when region ***R*** is revolved about the line y = 3.

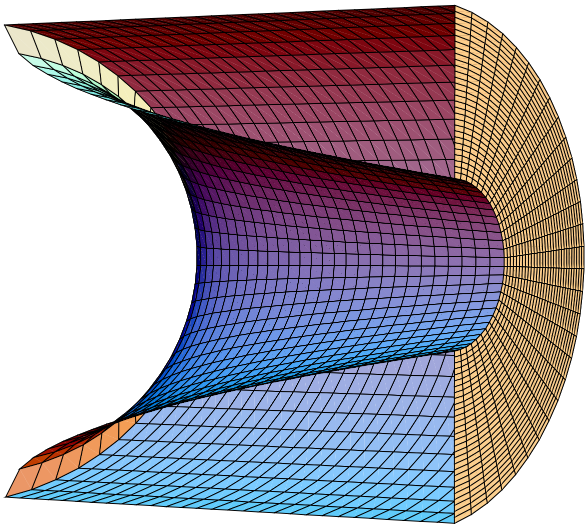



Note that, since the axis of revolution is not adjacent to region ***R***, there will be a hole in the solid.

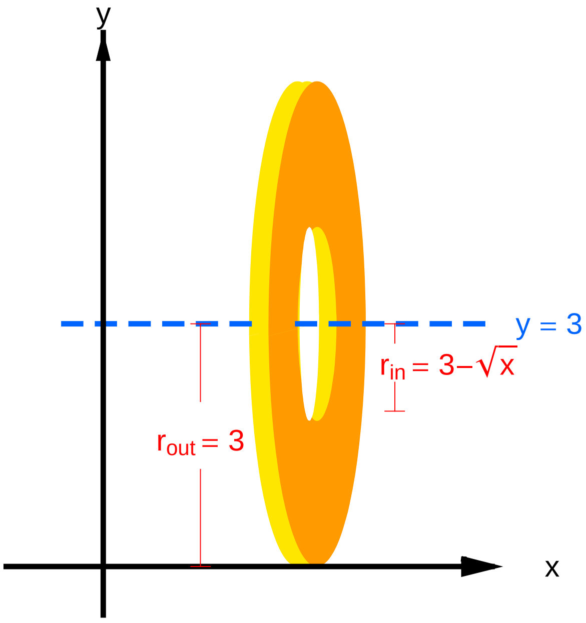
The solid of revolution looks like this:



and a cross-section looks like this:



When our rectangular region (see original graph) is rotated about the line y = 3 we get a disk with a hole in it, or a washer. To compute the volume of this washer we will find the volume of the "outside" disk and subtract the volume of the "inside" disk from it. The washer is depicted below:




The "outside" radius is the distance from the x-axis to the axis of revolution, or 3, and the "inside" radius is the difference of 3 and $\sqrt{x}$ or 3 - $\sqrt{x}$. The height of both disks is **Δx.** So the volume of the washer is given by

$$V_{\text{washer}} = V_{\text{outside disk}} - V_{\text{inisde disk}} = \pi\, r_{\text{out}}{}^2\, h - \pi\, r_{\text{in}}{}^2\, h$$

$$= \pi\,(3)^2\,\Delta\text{x} - \pi\left(3-\sqrt{x}\right)^2\Delta\text{x} \quad \text{or}$$

$$\pi\left[9-\left(3-\sqrt{x}\right)^2\right]\Delta\text{x}$$

If we approximate the volume of the solid with a finite number of washers "stacked" next to each other we get

$$V_{\text{solid}} \approx \sum_{i=1}^{n} \pi\left[9-\left(3-\sqrt{x}\right)^2\right]\Delta\text{x}$$

The exact volume of the solid can be found by adding up an infinite number of infinitely thin washers on the interval [0, 4]. So, if we take the limit of the Riemann sum above as the number of washers goes to infinity (or as **Δx** goes to zero) we obtain the following definite integral.

$$V_{\text{solid}} = \lim_{n\to\infty} \sum_{i=0}^{n} \pi\left[9-\left(3-\sqrt{x}\right)^2\right]\Delta\text{x} = \pi\int_0^4\left[9-\left(3-\sqrt{x}\right)^2\right]dx$$

Evaluating this integral using the FTC we obtain

$$\pi\int_0^4\left[9-\left(3-\sqrt{x}\right)^2\right]dx = \pi\int_0^4\left[9-\left(9-6\sqrt{x}-x\right)\right]dx = \pi\int_0^4\left(x+6\sqrt{x}\right)dx =$$

$$\pi\left(\frac{x^2}{2}+6\,\frac{x^{3/2}}{3/2}\right)\Bigg|_0^4 = \pi\left[\frac{4^2}{2}+4\cdot 4^{3/2}-(0+0)\right] = \pi\mathbf{[}\,8+32] = 40\,\pi$$